Online Appendix for "Chained Credit Contracts and Financial Accelerators"

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A The Detail of the Model

A.1 Credit Market

The participants of the credit markets include the FIs, the entrepreneurs, and the investors. Figures A and B illustrate how credit contracts are made among the participants and how our credit markets differ from the model of BGG (1999).

A.1.1 FE contract

Basic setting

The FE contract is made between an FI and a continuum of the risk-neutral entrepreneurs. In period t, each type i FI offers a loan contract to an infinite number of group j_i entrepreneurs.[1] An entrepreneur in group j_i owns net worth $N_{E,j_i}(s^t)$ and purchases capital of $Q(s^t) K_{j_i}(s^t)$, where s^t is the whole history of states until period t, $Q(s^t)$ is the price paid per unit of capital, and $K_{j_i}(s^t)$ is the quantity of capital purchased by the group j_i entrepreneur. Since the net worth $N_{E,j_i}(s^t)$ of the entrepreneur is smaller than the amount of the capital purchase $Q(s^t) K_{j_i}(s^t)$, the entrepreneur raises the rest of the funds $Q(s^t) K_{j_i}(s^t) - N_{E,j_i}(s^t)$ from the type i FI. The net return to a capital of a group j_i entrepreneur is a product of the two elements: an aggregate return to capital $R_E(s^{t+1})$ and an idiosyncratic productivity shock $\omega_{E,j_i}(s^{t+1})$, which we call the entrepreneurs' riskiness hereafter, that is specific to the group j_i entrepreneur.[2] There is informational asymmetry between lenders and borrowers in the FE contract and the FI cannot observe the realization of the idiosyncratic shock $\omega_{E,j_i}(s^{t+1})$ without paying the monitoring cost μ_E. Under this credit friction, the FE contract specifies:

- the amount of debt that the group j_i entrepreneur borrows from a type i FI, $Q(s^t) K_{j_i}(s^t) - N_{E,j_i}(s^t)$, and
- a cut-off value of idiosyncratic productivity shock $\omega_{E,j_i}(s^{t+1})$, which we denote by $\overline{\omega}_{E,j_i}(s^t)$, such that the group j_i entrepreneur is able to repay all of its debt if $\omega_{E,j_i}(s^{t+1}) \geq \overline{\omega}_{E,j_i}(s^t)$ and declares the default if $\omega_{E,j_i}(s^{t+1}) < \overline{\omega}_{E,j_i}(s^t)$.

[1] We assume that the bankruptcy cost associated with a direct credit contract between the investors and the enptrepreneurs is high enough so that there is no rational such contracts are made. By the similar assumption, a contract made between a type i FI and group j_{i^*} entrepreneurs for $i \neq i^*$ are left out from our analysis.

[2] Here, $\omega_{E,j_i}(s^t)$ is a unit mean, lognormal random variable distributed independently over time and across entrepreneurs. We express its density function by $f_E(\omega_E)$, and its cumulative distribution function by $F_E(\omega_E)$.

Entrepreneurs' participation constraint

Based on the FE contract, a portion $\int_{\overline{\omega}_{E,j_i}(s^t)}^{\infty} dF_E(\omega_E)$ of the entrepreneurs do not default and the rest of them default. *Ex post*, a non-default entrepreneur j_i receives the following net return to its capital holdings

$$\left(\omega_{E,j_i}\left(s^{t+1}\right) - \overline{\omega}_{E,j_i}\left(s^t\right)\right) R_E\left(s^{t+1}\right) Q\left(s^t\right) K_{j_i}\left(s^t\right).$$

The entrepreneurial loan rate $Z_{E,j_i}(s^{t+1})$ is therefore given by

$$Z_{E,j_i}\left(s^{t+1}\right) \equiv \frac{\overline{\omega}_{E,j_i}\left(s^t\right) R_E\left(s^{t+1}\right) Q\left(s^t\right) K_{j_i}\left(s^t\right)}{Q\left(s^t\right) K_{j_i}\left(s^t\right) - N_{E,j_i}\left(s^t\right)}. \tag{1}$$

A group j_i entrepreneur joins the FE contract only when the return from the credit contract is at least equal to its opportunity cost.[3] Instead of participating in the credit contract, a group j_i entrepreneur can purchase capital goods using only its own net worth $N_{E,j_i}(s^t)$. In this case, *ex ante,* the entrepreneur expects to receive the earning $\sum_{s^{t+1}} \Pr(s^{t+1}) R_E(s^{t+1}) N_{E,j_i}(s^t)$, where $\Pr(s^{t+1})$ is probability weight attached to state s^{t+1}. *Ex post* it receives the earning $\omega_{E,j_i}(s^{t+1}) R_E(s^{t+1}) N_{E,j_i}(s^t)$. Therefore, an FE contract between an FI and group j_i entrepreneurs is agreed by the entrepreneurs, only when the following inequality is expected to hold:

$$\begin{aligned} &\sum_{s^{t+1}} \Pr\left(s^{t+1}\right) \; R_E\left(s^{t+1}\right) Q\left(s^t\right) K_{j_i}\left(s^t\right) \left(\int_{\overline{\omega}_{E,j_i}(s^t)}^{\infty} \left(\omega_E - \overline{\omega}_{E,j_i}\left(s^t\right)\right) dF\left(\omega_E\right) \right) \\ &\geq \sum_{s^{t+1}} \Pr\left(s^{t+1}\right) R_E\left(s^{t+1}\right) N_{E,j_i}\left(s^t\right) \text{ for } \forall j_i. \end{aligned} \tag{2}$$

The left-hand side of inequality (2) is the expected return from the FE contract, and the right-hand side of inequality (2) is the expected return from investing the entrepreneurial current net worth $N_{E,j_i}(s^t)$ without joining the credit contract. Here, it is worth commenting that the above condition does not mean that entrepreneurs make no profit. Since $\sum_{s^{t+1}} \Pr(s^{t+1}) R_E(s^{t+1}) > 1$, entrepreneurs make positive profits on average, and accumulate their net worth that serves as a source of the financial accelerator mechanism. In the analysis below, we focus on the equilibrium where equation (2) holds with equality in all realizations of states.

FIs' profit from the FE contract

The expected earnings of the type i FI from an FE contract are obtained:

[3] Unlike the BGG (1999) and a number of extended models, the entrepreneur in our model is not an agent who optimizes the credit contract. Thus, we need to consider its participation constraint, which is the novelty of our model.

$$\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)\left(\sum_{s^{t+1}}\Pr\left(s^{t+1}\right)R_E\left(s^{t+1}\right)\right)Q\left(s^t\right)K_{j_i}\left(s^t\right),$$

where $\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)$ is the share of profits going to the lender:

$$\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)\equiv\int_{\overline{\omega}_{E,j_i}(s^t)}^{\infty}\overline{\omega}_{E,j_i}\left(s^t\right)dF_E\left(\omega_E\right)+\left(1-\mu_E\right)\int_0^{\overline{\omega}_{E,j_i}(s^t)}\omega_E dF_E\left(\omega_E\right). \quad (3)$$

The first term of $\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)$ illustrates the FIs' earnings when entrepreneurs do not default. When $\omega_{E,j_i}\left(s^{t+1}\right)\geq\overline{\omega}_{E,j_i}\left(s^t\right)$, a group j_i entrepreneur does not declare the default, and a type i FI receives the promised $\overline{\omega}_{E,j_i}\left(s^t\right)$ multiplied by $R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right)$. The second term of $\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)$ illustrates the FIs' earnings when entrepreneurs declare the default. When $\omega_{E,j_i}\left(s^{t+1}\right)<\overline{\omega}_{E,j_i}\left(s^t\right)$. a group j_i entrepreneur declares the default, and a type i FI monitors it by paying $\mu_E\omega_{E,j_i}\left(s^{t+1}\right)$ multiplied by $R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right)$. The type i FI then collects all of the entrepreneur's earnings, $\omega_{E,j_i}\left(s^{t+1}\right)$ multiplied by $R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right)$.

For the convenience of analysis below, we define *ex post* return on the loans from a type i FI to the group j_i entrepreneurs, $R_F\left(s^{t+1}\right)$, as

$$\int_{j_i}\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right)dj_i$$

$$\equiv R_F\left(s^{t+1}\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_{E,i}\left(s^t\right)\right)\text{ for }\forall s^{t+1}|s^t, \quad (4)$$

where

$$K_i\left(s^t\right)\equiv\int_{j_i}K_{j_i}\left(s^t\right)dj_i,\ \ N_{E,i}\left(s^t\right)\equiv\int_{j_i}N_{E,j_i}\left(s^t\right)dj_i.$$

The left-hand side of equation (4) represents the earnings that a type i FI receives from a continuum number of the FE contracts with group j_i entrepreneurs, and $Q\left(s^t\right)K_i\left(s^t\right)-N_{E,i}\left(s^t\right)$ represents the total amount of loans lent to the group j_i entrepreneurs.

A.1.2 IF contract

Basic setting

The IF contract is made between an investor and a continuum of the FIs. In period t, each type i FI holds the net worth $N_{F,i}\left(s^t\right)$ and makes loans to group j_i entrepreneurs at an amount of $Q\left(s^t\right)K_i\left(s^t\right)-N_{E,i}\left(s^t\right)$. Since the FI's net worth is smaller than its loans to the entrepreneurs, it borrows the rest $Q\left(s^t\right)K_i\left(s^t\right)-N_{F,i}\left(s^t\right)-N_{E,i}\left(s^t\right)$ from the investor. Each type i FI faces an idiosyncratic productivity shock $\omega_{F,i}\left(s^{t+1}\right)$, which we call the FIs' riskiness. This shock $\omega_{F,i}(s^{t+1})$ represents the technological differences across the FIs, for example, those associated with risk management, the maturity mismatch control, and loan securitization.[4] Consequently, the FI's receipt from the loans to the entrepreneurs is given by $\omega_{F,i}(s^{t+1}) R_F(s^{t+1}) (Q(s^t) K_i(s^t) - N_{E,i}(s^t))$.[5] Similarly to the FE contract, there is informational asymmetry between the lender and the borrowers in the IF contract and the investor can observe the realization of the shock $\omega_{F,i}(s^{t+1})$ only by paying the monitoring cost captured by μ_F. Under this environment, the IF contract specifies:[6]

- the amount of debt that a type i FI borrows from the investor, $Q(s^t) K_i(s^t) - N_{E,i}(s^t) - N_{F,i}(s^t)$, and
- a cut-off value of idiosyncratic shock $\omega_{F,i}(s^{t+1})$, which we denote by $\overline{\omega}_{F,i}(s^{t+1})$, such that the FI is able to repay all of its debt if $\omega_{F,i}(s^{t+1}) \geq \overline{\omega}_{F,i}(s^{t+1})$ and declares the default if $\omega_{F,i}(s^{t+1}) < \overline{\omega}_{F,i}(s^{t+1})$.

FIs' profit from the two credit contracts

According to the IF contract, a portion $\int_{\overline{\omega}_{F,i}(s^{t+1})}^{\infty} dF_F(\omega_F)$ of the FIs do not default while the rest of them default. The net profit of a non-default FI i equals its earnings from the FE contract multiplied by the idiosyncratic shock $\omega_{F,i}(s^{t+1})$ minus repayment to the investor:

$$\left(\omega_{F,i}\left(s^{t+1}\right) - \overline{\omega}_{F,i}\left(s^{t+1}\right)\right) R_F\left(s^{t+1}\right)\left(Q\left(s^t\right) K_i\left(s^t\right) - N_{E,i}\left(s^t\right)\right).$$

The FIs' loan rate $Z_{F,i}(s^{t+1})$ is therefore given by

$$Z_F\left(s^{t+1}\right) \equiv \frac{\overline{\omega}_{F,i}(s^{t+1}) R_F(s^{t+1}) (Q(s^t) K_i(s^t) - N_{E,i}(s^t))}{Q(s^t) K_i(s^t) - N_{F,i}(s^t) - N_{E,i}(s^t)}. \tag{5}$$

Investors' participation constraint

Investors participate in the IF contract only when they are better off by this. Note that they make repayment to the household from what they earn from the IF contract.

[4] Alternatively, one may interpret $\omega_{F,i}(s^t)$ as an idiosyncratic productivity shock that is specific to a group of firms, such as those in the same industry i or those located in the same region i, and interpret $\omega_{E,j_i}(s^t)$ as firm specific shock for those belonging to the same industry or region i. Suppose that there is an infinite number of industries (regions) that consist of an infinite number of firms and each type i FI lends funds to only one of the industries (regions) i. Industry-specific (region-specific) shock then affects the type i FI's earnings exclusively as if the FI is hit by an idiosyncractic productivity shock that is specific to the type i FI.

[5] Similarly to the entrepreneurial riskiness ω_{E,j_i}, the FIs' riskiness $\omega_{F,i}$ is a unit mean, lognormal random variable distributed independently over time and across FIs i. Its density function and its cumulative distribution function are given by $f_F(\omega_F)$ and $F_F(\omega_F)$, respectively.

[6] Similarly to BGG (1999), the contents of the FI contracts, the cut-off value $\overline{\omega}_{F,j_i}\left(s^{t+1}\right)$ is contingent on the aggregate states. By contrast, because of the structure of equation (2), the cut-off value $\overline{\omega}_{E,j_i}(s^t)$ is not contingent on aggregate states.

Denoting the risk-free rate of return in the economy by $R\left(s^t\right)$, the investor's net receipt from the IF contracts must at least equal the return from the risk-free investment. That is,

$$\Phi_F\left(\overline{\omega}_{F,i}\left(s^{t+1}\right)\right)R_F\left(s^{t+1}\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_{E,i}\left(s^t\right)\right)$$

$$\geq R\left(s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_{F,i}\left(s^t\right)-N_{E,i}\left(s^t\right)\right) \text{ for } \forall i, s^{t+1}, \tag{6}$$

where $\Phi_F\left(\overline{\omega}_{F,i}\left(s^{t+1}\right)\right)$ is the share of profits going to the lender:

$$\Phi_F\left(\overline{\omega}_{F,i}\left(s^{t+1}\right)\right)\equiv\overline{\omega}_{F,i}\left(s^{t+1}\right)\int_{\overline{\omega}_{F,i}\left(s^{t+1}\right)}^{\infty}dF_F\left(\omega_F\right)+\left(1-\mu_F\right)\int_0^{\overline{\omega}_{F,i}\left(s^{t+1}\right)}\omega_F dF_F\left(\omega_F\right). \tag{7}$$

Similarly to BGG (1999), we assume that the IF contract is contingent on aggregate states and the participation constraint (6) holds with equality state by state. Because investors face perfect competition, at the equilibrium, their earnings from the IF contracts equal to the amount of repayment to households in every state of the economy.

A.1.3 Optimal credit contract

Given the structure of the FE and the IF contract, a type i FI optimally chooses $K_{j_i}\left(s^t\right)$, $\overline{\omega}_{E,j_i}\left(s^t\right)$, $K_i\left(s^t\right)$, and $\overline{\omega}_{F,i}\left(s^{t+1}\right)$. The expected profit of a type i FI is given by

$$\sum_{s^{t+1}}\Pr\left(s^{t+1}\right)\left(\int_{\overline{\omega}_{F,i}\left(s^{t+1}\right)}^{\infty}\left(\omega_{F,i}-\overline{\omega}_{F,i}\left(s^{t+1}\right)\right)dF_F\left(\omega_F\right)\right)$$
$$\times R_F\left(s^{t+1}\right)\left(Q\left(s^t\right)K_i\left(s^t\right)-N_{E,i}\left(s^t\right)\right). \tag{8}$$

The FI maximizes the term (8), subject to the investor's participation constraint (6) and entrepreneurial participation constraint (2) for all of the group j_i entrepreneurs. In practice, the terms of credit contracts specify loan rates such as Z_{E,j_i} and $Z_{F,j}$ rather than threshold idiosyncratic shocks' values $\bar{\omega}_{F,i}$ and $\bar{\omega}_{E,i}$. As shown in equations (1) and (5), choosing the threshold idiosyncratic shocks' values $\bar{\omega}_{F,i}$ and $\bar{\omega}_{E,i}$ is equivalent to choosing the loan rates $Z_{F,j}$ and Z_{E,j_i}.

The first-order condition is given by

$$
\begin{aligned}
0 = & \sum_{s^{t+1}} \Pr\left(s^{t+1}\right) \left\{ \left(1 - \Gamma_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)\right) \Phi_E\left(\overline{\omega}_E\left(s^t\right)\right) R_E\left(s^{t+1}\right) \right. \\
& + \frac{\Gamma'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)}{\Phi'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)} \Phi_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right) \Phi_E\left(\overline{\omega}_E\left(s^t\right)\right) R_E\left(s^{t+1}\right) \\
& - \frac{\Gamma'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)}{\Phi'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)} R(s^t) \\
& + \frac{\left\{1 - \Gamma_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right)\right\} \Phi'_E\left(\overline{\omega}_E\left(s^t\right)\right)}{\Gamma'_E\left(\overline{\omega}_E\left(s^t\right)\right)} \left(1 - \Gamma_E\left(\overline{\omega}_E\left(s^t\right)\right)\right) R_E\left(s^{t+1}\right) \\
& \left. + \frac{\Gamma'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right) \Phi_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right) \Phi'_E\left(\overline{\omega}_E\left(s^t\right)\right)}{\Phi'_F\left(\overline{\omega}_F\left(s^{t+1}\right)\right) \Gamma'_E\left(\overline{\omega}_E\left(s^t\right)\right)} \left(1 - \Gamma_E\left(\overline{\omega}_E\left(s^t\right)\right)\right) R_E\left(s^{t+1}\right) \right\} \\
& \text{for } \forall j_i, \qquad (9)
\end{aligned}
$$

where $\Gamma_k\left(\overline{\omega}_k\left(s^t\right)\right)$ for $k = \{F, E\}$ is given by

$$
\begin{aligned}
\Gamma_k\left(\overline{\omega}_k\left(s^t\right)\right) & \equiv \int_{\overline{\omega}_k(s^t)}^{\infty} \overline{\omega}_k\left(s^t\right) dF_k\left(\omega_k\right) + \int_0^{\overline{\omega}_k\left(s^t\right)} \omega_k dF_k\left(\omega_k\right) \\
& = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log \overline{\omega}_k\left(s^t\right) - 0.5\sigma_k^2}{\sigma_k}} \exp\left(-\frac{v_k^2}{2}\right) dv_k \\
& + \frac{\overline{\omega}_k\left(s^t\right)}{\sqrt{2\pi}} \int_{\frac{\log \overline{\omega}_k(s^t) + 0.5\sigma_k^2}{\sigma_k}}^{\infty} \exp\left(-\frac{v_k^2}{2}\right) dv_k.
\end{aligned}
$$

This first-order condition illustrates the relationship between the external finance premium $\sum_{s^{t+1}} \Pr\left(s^{t+1}\right) R_E\left(s^{t+1}\right) / R\left(s^t\right)$ and the cut-off values $\overline{\omega}_F\left(s^{t+1}\right)$ and $\overline{\omega}_E\left(s^t\right)$. Equation (9) together with the two participation constraints, equations (2) and (6) characterizes the optimal terms of the two credit contracts.

The relationship between the two net worths $N_F\left(s^t\right)$ and $N_E\left(s^t\right)$ and the external finance premium $\sum_{s^{t+1}} \Pr\left(s^{t+1}\right) R_E\left(s^{t+1}\right) / R\left(s^t\right)$ is delivered by arranging equations (4) and (6):

$$\frac{\sum_{s^{t+1}} \Pr\left(s^{t+1}\right) R_E\left(s^{t+1}\right)}{R\left(s^t\right)} = \overbrace{\left(1 - \frac{N_F\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)} - \frac{N_E\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)}\right)}^{\text{[1] ratio of the debt to the size of the capital investment}}$$

$$\times \overbrace{\Phi_F\left(\overline{\omega}_F\left(\frac{N_F\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)}, \frac{N_E\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)}\right)\right)^{-1}}^{\text{[2] inverse of share of profit going to the investors in the IF contract}}$$

$$\times \overbrace{\Phi_E\left(\overline{\omega}_E\left(\frac{N_E\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)}\right)\right)^{-1}}^{\text{[3] inverse of share of profit going to the FIs in the FE contract}}$$

$$\equiv S\left(n_F\left(s^t\right), n_E\left(s^t\right)\right), \tag{10}$$

where[7]

$$n_F\left(s^t\right) \equiv \frac{N_F\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)} \text{ and } n_F\left(s^t\right) \equiv \frac{N_E\left(s^t\right)}{Q\left(s^t\right) K\left(s^t\right)}.$$

Since $R_E\left(s^{t+1}\right)$ equals the aggregate return to capital in the equilibrium, for given size of risk-free rate, a higher external finance premium implies that capital investment is more depressed. This premium is influenced by net worth to aggregate capital ratio in the two sectors through the three terms in equation (10): [1] the ratio of total debt to aggregate capital; [2] the share of profit in the IF contract going to the investors; and [3] the share of profit in the FE contract going to the FIs.

The term [1] represents the capital investment leverage from the investor's viewpoint. Since the two credit contracts are chained, a decline in either of the two net worths indicates a higher leverage to the investor. In compensation for the increase in the expected default probability, therefore, the investor requires a higher return from the investment regardless of the holder of the net worth. The terms [2] and [3] show how each net worth affects the external finance premium. When net worth deteriorates, the default probability increases, dropping the lenders' shares as indicated in equations (3) and (7) due to the increases in the default costs. Other things being equal, a reduction in the lenders' share needs to be met with an increase in the external finance premium so that the investor's participation constraint is maintained.

The terms [2] and [3] are pinned down by equations (2), (6), and (9). It is notable that the term [2] is affected by both of the two net worths to aggregate capital ratios, $n_F\left(s^t\right)$ and $n_E\left(s^t\right)$, and that the term [3] is affected only by the entrepreneurial net

[7] Notice that because the ratio of net worth in each sector to aggregate capital $n_F\left(s^t\right)$ and $n_E\left(s^t\right)$ are identical across types of the FIs and across the groups of entrepreneurs, subscripts i and j_i are both dropped from the expressions. Consequently, similarly to BGG (1999), the developments of these ratios for individual FI and the entrepreneur are tracked by the ratio of aggregate net worth in each sector to aggregate capital.

worth to aggregate capital ratio $n_E(s^t)$. Accordingly, marginal changes in the two sectors' net worth bring about a different size of impact on the external finance premium. Consequently, the distribution of the net worth across the FIs and the entrepreneurs has a significant implication for the investment and the aggregate economy. To see this more in detail, in the following section, we investigate the property of $S(n_F(s^t), n_E(s^t))$ both analytically and numerically.

A.1.4 Cost-of-Funds Curve

Role of the FIs' net worth and the entrepreneurial net worth

The external finance premium $\sum_{s^{t+1}} \Pr(s^{t+1}) R_E(s^{t+1}) / R(s^t)$ is negatively related to the net worth in each of the borrowing sectors. That is, the cost-of-funds curve (10) is downward sloping with respect to net worth to aggregate capital ratios:

$$\frac{\partial S(n_F(s^t), n_E(s^t))}{\partial n_F(s^t)} < 0 \quad \text{and}$$
$$\frac{\partial S(n_F(s^t), n_E(s^t))}{\partial n_E(s^t)} < 0. \tag{11}$$

We provide the analytical proof in the next section. This property is the same as that in the model of BGG (1999), although there are two arguments n_F and n_E in our model, while there is only one argument n_E in BGG (1999).

For illustrative purposes, we also conduct numerical computation regarding how the external finance premium and the expected default costs vary with a size of one sector's net worth ratio, keeping one other's net worth ratio unchanged.[8,9] Figure 1 in the main paper indicates that the external finance premium is decreasing in both of the FIs' net worth to capital ratio (left panel) and the entrepreneurial net worth to capital ratio (right panel). As shown in the second term of equations (3) and (7), expected default costs, the term multiplied by monitoring cost μ_E and μ_F, are the key determinants of the lenders shares in equation (10), affecting the external finance premium. Figure C shows that, when a size of capital investment becomes large relative to entities' net worth, which implies a smaller net worth to capital ratio, the expected default cost of

[8] In this section, unless otherwise noted, we set the model parameters pertaining to the two credit contracts following BGG (1999) for comparison purpose. The values for parameters μ_E, σ_E, and $1-\gamma_E$ are set equal to the values of the monitoring cost, the entrepreneurial riskiness and the entrepreneurial death rate adopted in BGG (1999), respectively. It is assumed that $\mu_F = \mu_E$, $\sigma_F = \sigma_E$, and $\gamma_F = \gamma_E$ so that the two credit contracts are symmetric in terms of these parameters. In the following section, we calibrate the model to the actual data of the U.S. economy and explore the quantitative aspects of the model.

[9] Note that this exercise captures one-time snap shot of the outcomes on the IF and FE contracts of a given amount of net worth $N_F(s^t)$ and $N_E(s^t)$. We discuss dynamic apsects of the model in sections below.

the corresponding credit contract elevates, leading to a higher external finance premium through the term [2] or [3] in equation (10). In addition, since a shortage of the two entities' net worth implies a higher investment leverage for the investors, the term [1] also helps increase the external finance premium. FIs' net worth to capital ratio has no effect on the default cost in the FE contract, as the top-right panel shows in Figure C. This comes from the participation constraint for entrepreneurs (2) that is independent of $n_F(s^t)$.

The roles of the two net worths in determining the external finance premium are quantitatively different. This is because a change in the FIs' net worth to capital ratio has no influence on the expected default cost of the FE contract, while a change in the entrepreneurial net worth to capital affects the expected default cost of both contracts, as shown in Figure C. Because the entrepreneur's participation constraint is independent of the FI's net worth while the investor's participation constraint is affected by the entrepreneurial net worth, their impacts on the default probability becomes asymmetric. Consequently, other things being equal, an increase in the entrepreneur's net worth reduces the external finance premium more than does the FIs' net worth.[10]

Role of the distribution of net worth across sectors

We next discuss the implication of the distribution of net worth for the external finance premium. In contrast to BGG (1999) in which only the entrepreneurial net worth is studied, our model consists of the two distinct net worths distributed to the FIs and the entrepreneurs. Since the two net worths are not substitutable across sectors and affect the terms of the two credit contracts differently, the relative size of each net worth is important for the external finance premium.

To see this distributional aspect of the model in detail, we display how the FIs' share in the net worth alters the external finance premium in Figure 2 in the main text. The share of the net worth held by the FI sector is depicted on the horizontal axis and the corresponding external finance premium is depicted on the vertical axis. We set the ratio of total net worth to the total amount of capital investment $(N_E(s^t) + N_F(s^t)) / (Q(s^t) K(s^t))$ equals to 0.6. We first concentrate our analysis on the case where the technology of the financial intermediation and the extent of the borrowers' heterogeneity are identical across the two credit contracts. That is, parameter values for the monitoring costs and riskiness are the same, namely, $\mu_F = \mu_E$ and $\sigma_F = \sigma_E$. The result under this symmetric assumption is reported by the solid lines in each figure.

The U-shaped cost-of-funds curve in Figure 2 indicates that a net worth disruption in a sector with a lower net worth causes a disproportionately large increase in the external

[10] As we demonstrate below, however, our calibration based on the U.S. economy suggests that the size of net worth and the technologies associated with the credit contracts are asymmetric across the two borrowing sectors. Consequently, despite the argument here, the FIs' net worth plays a disproportionately large effect on the external finance premium.

finance premium. In other word, a large discrepancy between the size of FIs' net worth and that of the entrepreneurial net worth aggravates the condition of external finance, which dampens aggregate investment. By contrast, when the two sectors' net worths are distributed more evenly, the external finance premium is maintained at a low level, encouraging aggregate investment. The two net worths thus work complementarily in reducing the external finance premium. This complementarity of the net worths makes a sharp contrast with the existing related models, like Holmstrom and Tirole (1997), where the distribution of the net worth plays no role in determining the external finance premium in the economy.

Figure D illustrates the same points from the viewpoints of the expected default costs. When the FIs' net worth is relatively scarce, for instance $N_F(s^t)/(N_F(s^t)+N_E(s^t)) = 0.2$, a unit transfer of the net worth from the entrepreneurs to the FIs lowers the expected default cost of the IF contract significantly, raising that of the FE contract only moderately. Similarly, when the entrepreneurial net worth is relatively scarce, for instance, $N_F(s^t)/(N_F(s^t)+N_E(s^t)) = 0.8$, the same transfer lowers the expected default cost of the IF contract only moderately, raising that of the FE contract significantly. Because of these non-linearities, the external finance premium becomes highly sensitive to a change in net worth in a sector that possesses a relatively scarce net worth.

These properties are affected by the technology parameters and borrowers' heterogeneity parameters associated with the two credit contracts. In Figures 2 and D, the lines with black circles and the dotted lines display the cost-of-funds curve and the expected default costs when the monitoring costs are set to $\mu_F = \mu_E/2 = \mu/2$, and $\mu_F/2 = \mu_E = \mu/2$, respectively. In the former economy, for example, because the FIs' default is less costly, a scarcity in the FIs' net worth leads to a limited rise in the external finance premium. Consequently, the cost-of-funds curve is shifted downwards and tilted to the left. A similar mechanism is at work in the latter economy.

In Figures 3 and E, the lines with black circles and the dotted lines display the cost-of-funds curve and the expected default costs when the riskiness is set to $\sigma_F = \sigma_E/2 = \sigma/2$, and $\sigma_F/2 = \sigma_E = \sigma/2$, respectively. Similarly to the consequence of reducing monitoring costs, the decrease in the riskiness lowers the default cost and shifts the cost-of-funds curve downwards. Because the credit friction stemming from the information asymmetry is moderated in the IF contract, the default cost of the FIs in the IF contract falls, shifting the bottom of the U-shape to the left.

A.1.5 Dynamic Behavior of Net Worth

The net worth of the FIs and the entrepreneurs, $N_F(s^t)$ and $N_E(s^t)$, evolves depending on their profits from the credit contracts and their labor income. Both FIs and entrepreneurs inelastically supply a unit of labor to final goods producers and receive labor income $W_F(s^t)$ and $W_E(s^t)$.[11] The aggregate net worths of the FIs and the entrepreneurs

[11] See BGG (1999) and CMR (2008) for the technical reason behind this specification.

are given by

$$N_F\left(s^{t+1}\right)=\gamma_F V_F\left(s^t\right)+W_F\left(s^t\right)+\varepsilon_{N_F}\left(s^t\right), \tag{12}$$

$$N_E\left(s^{t+1}\right)=\gamma_E V_E\left(s^t\right)+W_E\left(s^t\right)+\varepsilon_{N_E}\left(s^t\right), \tag{13}$$

with their profits:

$$V_F\left(s^t\right)\equiv\left(\int_{\overline{\omega}_F\left(s^{t+1}\right)}^{\infty}\left(\omega_F-\overline{\omega}_F\left(s^{t+1}\right)\right)dF_F\left(\omega_F\right)\right)\Phi_E\left(\overline{\omega}_E\left(s^t\right)\right)R_E\left(s^{t+1}\right)Q\left(s^t\right)K\left(s^t\right),$$

$$V_E\left(s^t\right)\equiv\left(\int_{\overline{\omega}_E\left(s^t\right)}^{\infty}\left(\omega_E-\overline{\omega}_E\left(s^t\right)\right)dF_E\left(\omega_E\right)\right)R_E\left(s^{t+1}\right)Q\left(s^t\right)K\left(s^t\right).$$

Here, γ_F and γ_E are exogenous probabilities that each FI and entrepreneur survives to the next period. The FIs and the entrepreneurs who are in business in period t and fail to survive in period $t+1$ consume $\left(1-\gamma_F\right)V_F\left(s^t\right)$ and $\left(1-\gamma_E\right)V_E\left(s^t\right)$, respectively.

The net worth accumulations in both sectors are affected by exogenous shocks represented by $\varepsilon_{N_F}\left(s^t\right)$ and $\varepsilon_{N_E}\left(s^t\right)$ that are orthogonal to the fundamental earnings from the credit contracts. We assume these shocks are i.i.d. They may indicate financial shocks that capture an "asset bubble," "irrational exuberance," or an "innovation in the efficiency of credit contracts," hitting the FI sector or the entrepreneurial sector.[12]

A.2 The Rest of the Economy

The setup for the rest of the economy is standard, following the BGG.

Household

A representative household is infinitely lived, and maximizes the following utility function:

$$\max_{C\left(s^{t+l}\right),H\left(s^{t+l}\right),D\left(s^{t+l}\right)}\sum_{l=0}^{\infty}\beta^{t+l}\sum_{s^{t+l}}\Pr\left(s^{t+l}\right)\left\{\log C\left(s^{t+l}\right)-\chi\frac{H\left(s^{t+l}\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right\}, \tag{14}$$

subject to

$$C\left(s^{t+l}\right)+D\left(s^{t+l}\right)\leq W\left(s^{t+l}\right)H\left(s^{t+l}\right)+R\left(s^{t+l-1}\right)D\left(s^{t+l-1}\right)+\Pi\left(s^{t+l}\right)-T\left(s^{t+l}\right),$$

[12] The setting of these net worth shocks is borrowed from Gilchrist and Leahy (2002). See also CMR (2008) and Nolan and Thoenissen (2009) for the interpretation of these net worth shocks under credit market imperfection. In these studies, the exit ratio of entrepreneurs γ_E, that is analogous to γ_E in the equation (13), obeys the stochastic law of motion, generating an unexpected change in the entrepreneurial net worth.

where $C(s^t)$ is final goods consumption, $H(s^t)$ is hours worked, $D(s^t)$ is real deposits, $W(s^t)$ is the real wage measured by the final goods, $R(s^{t-1})$ is the real risk-free rate from the deposit $D(s^t)$ between time $t-1$ and t, and $T(s^t)$ is the lump-sum transfer. $\beta \in (0,1)$, η and χ are the subjective discount factor, the elasticity of leisure, and the utility weight on leisure. The first-order conditions associated with the household's maximization problem are given by

$$\frac{1}{C(s^t)} = \beta \sum_{s^{t+1}} \Pr\left(s^{t+1}\right)\left\{\frac{1}{C(s^{t+1})} R\left(s^{t+1}\right)\right\}, \tag{15}$$

$$W\left(s^t\right) = \chi H\left(s^t\right)^{\frac{1}{\eta}} C\left(s^t\right). \tag{16}$$

Final Goods Producers

Final goods producers are price takers in both input markets and output markets. They hire three types of labor inputs: $H(s^t)$, $H_F(s^t)$, and $H_E(s^t)$, from the household, the FIs, and the entrepreneurs, and pay real wages $W(s^t)$, $W_F(s^t)$, and $W_E(s^t)$ to each type of labor inputs, respectively. They rent capital $K(s^{t-1})$ from the entrepreneurs with a rental price $R_E(s^t)$ in the beginning of each period and return it to the entrepreneurs at the end of each period. The maximization problem of the final goods producers is given by

$$\begin{aligned}\max_{Y(s^t),K(s^{t-1}),H(s^t),H_F(s^t),H_E(s^t)} & Y\left(s^t\right) + Q\left(s^{t-1}\right) K\left(s^{t-1}\right)(1-\delta) \\ & - R_E\left(s^t\right) Q\left(s^{t-1}\right) K\left(s^{t-1}\right) - W\left(s^t\right) H\left(s^t\right) \\ & - W_F\left(s^t\right) H_F\left(s^t\right) - W_E\left(s^t\right) H_E\left(s^t\right),\end{aligned}$$

subject to

$$\begin{aligned} Y\left(s^t\right) &= A \exp\left(e^A\left(s^t\right)\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha}, \text{ and} \\ L\left(s^t\right) &\equiv \left(H\left(s^t\right)\right)^{1-\Omega_E-\Omega_F}\left(H_F\left(s^t\right)\right)^{\Omega_F}\left(H_E\left(s^t\right)\right)^{\Omega_E}, \end{aligned} \tag{17}$$

where $Y(s^t)$ is the final goods produced and $A\exp(e_A(s^t))$ is the level of total factor productivity (TFP). $\delta \in (0,1]$, α, Ω_E, and Ω_F are the depreciation rate of capital goods, the capital share, the share of the FIs' labor inputs, and the share of the entrepreneurial labor inputs. We assume that TFP evolves following the equation below:

$$e_A\left(s^t\right) = \rho_A e_A\left(s^{t-1}\right) + \varepsilon_A\left(s^t\right), \tag{18}$$

where $\rho_A \in (0,1)$ is the autoregressive root of the TFP and $\varepsilon_A(s^t)$ is the exogenous shock that is normally distributed with mean zero.

The first-order conditions of the final goods producers are

$$\alpha \frac{Y\left(s^{t}\right)}{K\left(s^{t-1}\right)} - R_{E}\left(s^{t}\right) Q\left(s^{t-1}\right) + Q\left(s^{t-1}\right)(1-\delta) = 0, \tag{19}$$

$$(1-\alpha)\left(1-\Omega_{F}-\Omega_{E}\right) \frac{Y\left(s^{t}\right)}{H\left(s^{t}\right)} = W\left(s^{t}\right), \tag{20}$$

$$(1-\alpha)\,\Omega_{F} \frac{Y\left(s^{t}\right)}{H_{F}\left(s^{t}\right)} = W_{F}\left(s^{t}\right), \text{ and} \tag{21}$$

$$(1-\alpha)\,\Omega_{E} \frac{Y\left(s^{t}\right)}{H_{E}\left(s^{t}\right)} = W_{E}\left(s^{t}\right). \tag{22}$$

Capital Goods Producers

Capital goods producers own technology that converts final goods to capital goods. They sell capital goods to the entrepreneurs in a competitive market with price $Q\left(s^{t}\right)$. In the beginning of the period t, the capital goods producers purchase $I\left(s^{t}\right)$ amount of final goods from final goods producers. They also receive $K\left(s^{t-1}\right)(1-\delta)$ of used capital goods from the entrepreneurs at price $Q\left(s^{t-1}\right)$. They then produce capital goods $K\left(s^{t}\right)$, using technology F_{I}. The capital goods producers' problem is to maximize the profit function given below:

$$\max_{I_t} \sum_{l=0}^{\infty} \sum_{s^{t+l}} \Pr\left(s^{t+l}\right) \Lambda_{t,t+l}(s^{t+l}) \cdot \left[Q\left(s^{t+l}\right) K\left(s^{t+l}\right) - (1-\delta)\, Q\left(s^{t+l}\right) K\left(s^{t+l-1}\right) - I\left(s^{t+l}\right)\right], \tag{23}$$

where $\Lambda_{t,t+l}(s^{t+l}) \equiv \beta C\left(s^{t}\right)/C\left(s^{t+l}\right)$ is a stochastic discount factor and F_{I} is defined as follows:

$$F_{I}\left(I\left(s^{t+l}\right), I\left(s^{t+l-1}\right)\right) \equiv \frac{\kappa}{2}\left(\frac{I\left(s^{t+l}\right)}{I\left(s^{t+l-1}\right)} - 1\right)^{2}.$$

Note that κ is a parameter associated with investment adjustment cost.[13]

Because capital depreciates in each period, the evolvement of total capital available in period t is given by

$$K\left(s^{t}\right) = \left(1 - F_{I}\left(I\left(s^{t}\right), I\left(s^{t-1}\right)\right)\right) I\left(s^{t}\right) + (1-\delta)\, K\left(s^{t-1}\right). \tag{24}$$

[13] A term for used capital $K\left(s^{t-1}\right)$ sold by the entrepreneurs at the end of the period $t-1$ does not appear in equation (23). This is because, following BGG (1999), we assume that the price of capital that the entrepreneurs sell back to the capital goods producers, say $\overline{Q}\left(s^{t}\right)$, is close to the price of newly produced capital $Q\left(s^{t}\right)$ around the steady state.

Government

The government collects a lump-sum tax from a household $T(s^t)$, and spends $G(s^t)$. A balanced budget is maintained in each period t as

$$G\left(s^t\right) = T\left(s^t\right). \tag{25}$$

Resource Constraint

The resource constraint for final goods is written as

$$\begin{aligned} Y\left(s^t\right) &= C\left(s^t\right) + I\left(s^t\right) + G\left(s^t\right) \\ &+ \mu_F \left(\int_0^{\overline{\omega}_F\left(s^{t+1}\right)} \omega_F dF_F\left(\omega_F\right) \right) R_F\left(s^t\right) \left\{ Q\left(s^{t-1}\right) K\left(s^{t-1}\right) - N_E\left(s^{t-1}\right) \right\}. \\ &+ \mu_E \left(\int_0^{\overline{\omega}_E\left(s^t\right)} \omega_E dF_E\left(\omega_E\right) \right) R_E\left(s^t\right) Q\left(s^{t-1}\right) K\left(s^{t-1}\right) \\ &+ \left(1-\gamma_F\right) V_F\left(s^t\right) + \left(1-\gamma_E\right) V_E\left(s^t\right). \end{aligned} \tag{26}$$

The fourth and the fifth terms on the right-hand side of the equation represent the default costs spent by the investors and the FIs, respectively. The sixth and seventh terms represent the consumption of the FIs and the entrepreneurs who exit from the business in period t, respectively.

A.3 Equilibrium Condition

An equilibrium consists of a set of prices, $\{R(s^t),\ R_F(s^t),\ R_E(s^t),\ W(s^t),\ W_F(s^t),$ $W_E(s^t), Q(s^t), R_F(s^{t+1}), R_E(s^{t+1}), Z_F(s^{t+1}), Z_E(s^{t+1})\}_{t=0}^{\infty}$, and the allocations $\{\{\overline{\omega}_{F,i}(s^{t+1})\}_{i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{\overline{\omega}_{E,j_i}(s^t)\}_{j_i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{N_{F,i}(s^t)\}_{i=1}^{\infty}\}_{t=0}^{\infty}$, $\{\{N_{E,j_i}(s^t)\}_{j_i=1}^{\infty}\}_{t=0}^{\infty}$, $\{Y(s^t),\ C(s^t),\ D(s^t),$ $I(s^t),\ K(s^t),\ H(s^t),\ \Pi(s^t)\}\}_{t=0}^{\infty}$, for a given government policy $\{G(s^t), T(s^t)\}_{t=0}^{\infty}$, realization of exogenous variables $\{\varepsilon_A(s^t),\ \varepsilon_{N_F}(s^t), \varepsilon_{N_E}(s^t)\}_{t=0}^{\infty}$ and initial conditions $\{N_{F,i,-1}\}_{i=1}^{\infty}$, $\{N_{E,j_i,-1}\}_{j_i=1}^{\infty}$, $\{K_{-1}\}$ such that for all t, i, j_i and h :

(i) the household maximizes its utility given the prices;
(ii) the FIs maximize their profits given the prices;
(iii) the entrepreneurs maximize their profits given the prices;
(iv) final goods producers maximize their profits given the prices;
(v) capital goods producers maximize their profits given the prices;
(vi) the government budget constraint holds; and
(vii) markets clear.

B Proof of a Downward Slope of the Cost-of-Funds Curve

In this appendix, we provide the proof for inequality (11). As in BGG (1999), we neglect aggregate risks and express variables without state s^t. First, let us calculate the partial derivatives of Γ_k and Φ_k defined in Appendix A with respect to $\overline{\omega}_k$ for $k = \{F, E\}$. It is simple to show

$$\begin{aligned} \Gamma_k' &= \int_{\overline{\omega}_k}^{\infty} dF > 0, \;\; \Gamma_k'' = -\overline{\omega}_k < 0, \\ \Phi_k' &= \int_{\overline{\omega}_k}^{\infty} dF - \mu_k \overline{\omega}_k, \;\; \text{and } \Phi_k'' = -\overline{\omega}_k - \mu_k < 0. \end{aligned} \tag{27}$$

When $\overline{\omega}_k$ is lower than a certain threshold $\overline{\omega}_k^*$, then we have

$$\Phi_k' > 0. \tag{28}$$

We assume that this condition is satisfied as BGG (1999) show in Appendix A.1 in their paper.

Second, we analyze the partial derivative of $S(n_F, n_E) = R_E\left(\overline{\omega}_F(n_F, n_E), \overline{\omega}_E(n_F, n_E)\right)/R$ with respect to $\overline{\omega}_F$ and $\overline{\omega}_E$. The partial derivative of equation (9) with respect to $\overline{\omega}_F$ yields

$$\begin{aligned} -\frac{\Gamma_F'\Gamma_E' R}{R_E}\frac{\partial R_E}{\partial \overline{\omega}_F} &= (1-\Gamma_F)\Phi_F''\Phi_E\Gamma_E + \Gamma_F''\Gamma_E'\left(\Phi_F\Phi_E R_E - R\right) \\ &\quad + (1-\Gamma_F)\Phi_F''\Phi_E'(1-\Gamma_E)R_E + \Gamma_F''\Phi_F\Phi_E'(1-\Gamma_E)R_E. \end{aligned}$$

Eliminating R on the right-hand side of the equation using equation (9), we obtain

$$\begin{aligned} -\frac{\Gamma_F'\Gamma_E' R}{R_E}\frac{\partial R_E}{\partial \overline{\omega}_F} &= (1-\Gamma_F)\left(\Phi_E\Gamma_E' + \Phi_E'(1-\Gamma_E)\right)\left(\Phi_F'' - \frac{\Gamma_F''\Phi_F'}{\Gamma_F'}\right)R_E \\ &= (1-\Gamma_F)\left(\Phi_E\Gamma_E' + \Phi_E'(1-\Gamma_E)\right)\left(-\frac{\mu_F(\Gamma_F' + \overline{\omega}_F^2)}{\Gamma_F'}\right)R_E \\ &< 0, \end{aligned}$$

and in turn,

$$\frac{\partial R_E}{\partial \overline{\omega}_F} > 0. \tag{29}$$

Similarly, we can prove

$$\frac{\partial R_E}{\partial \overline{\omega}_E} > 0. \tag{30}$$

Third, we compute the partial derivatives of $\overline{\omega}_k$ with respect to n_k for $k = \{F, E\}$. The entrepreneurs' participation constraint (2) is written as

$$1 - \Gamma_E = n_E, \tag{31}$$

which yields

$$\frac{\partial \overline{\omega}_E}{\partial n_F} = 0 \text{ and } \frac{\partial \overline{\omega}_E}{\partial n_E} = -\frac{1}{\Gamma'_E}. \tag{32}$$

The investors' participation constraint (6) is given by

$$\Phi_F \Phi_E R_E = R(1 - n_F - n_E). \tag{33}$$

Its partial derivatives with respect to n_F and n_E read

$$\begin{aligned} \frac{\partial \overline{\omega}_F}{\partial n_F}\Phi'_F + \frac{\partial \overline{\omega}_E}{\partial n_F}\Phi'_E + \frac{\partial \overline{\omega}_F}{\partial n_F}\frac{\partial R_E}{\partial \overline{\omega}_F} + \frac{\partial \overline{\omega}_E}{\partial n_F}\frac{\partial R_E}{\partial \overline{\omega}_E} &= -R, \text{ and} \\ \frac{\partial \overline{\omega}_F}{\partial n_E}\Phi'_F + \frac{\partial \overline{\omega}_E}{\partial n_E}\Phi'_E + \frac{\partial \overline{\omega}_F}{\partial n_E}\frac{\partial R_E}{\partial \overline{\omega}_F} + \frac{\partial \overline{\omega}_E}{\partial n_E}\frac{\partial R_E}{\partial \overline{\omega}_E} &= -R. \end{aligned}$$

From (32), these two equations are simplified to

$$\begin{aligned} \frac{\partial \overline{\omega}_F}{\partial n_F}\left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right) &= -R, \text{ and} \\ \frac{\partial \overline{\omega}_F}{\partial n_E}\Phi'_F - \frac{1}{\Gamma'_E}\Phi'_E + \frac{\partial \overline{\omega}_F}{\partial n_E}\frac{\partial R_E}{\partial \overline{\omega}_F} - \frac{1}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E} &= -R. \end{aligned} \tag{34}$$

The former equation reveals

$$\frac{\partial \overline{\omega}_F}{\partial n_F} < 0 \tag{35}$$

from equation (29). Therefore, we can prove that

$$\frac{\partial R_E}{\partial n_F} = \frac{\partial \overline{\omega}_F}{\partial n_F}\frac{\partial R_E}{\partial \overline{\omega}_F} + \frac{\partial \overline{\omega}_E}{\partial n_F}\frac{\partial R_E}{\partial \overline{\omega}_E} = \frac{\partial \overline{\omega}_F}{\partial n_F}\frac{\partial R_E}{\partial \overline{\omega}_F} < 0.$$

The cost of curve decreases as n_F increases. The latter equation of (34) leads to

$$\frac{\partial \overline{\omega}_F}{\partial n_E} = \left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right)^{-1}\left(\frac{\Phi'_E}{\Gamma'_E} + \frac{1}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E} - R\right).$$

Inserting this into

$$\frac{\partial R_E}{\partial n_E} = \frac{\partial \overline{\omega}_F}{\partial n_E}\frac{\partial R_E}{\partial \overline{\omega}_F} + \frac{\partial \overline{\omega}_E}{\partial n_E}\frac{\partial R_E}{\partial \overline{\omega}_E},$$

we have

$$
\begin{aligned}
\frac{\partial R_E}{\partial n_E} &= \left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right)^{-1} \left(\frac{\Phi'_E}{\Gamma'_E} + \frac{1}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E} - R\right)\frac{\partial R_E}{\partial \overline{\omega}_F} - \frac{1}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E} \\
&= \left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right)^{-1} \left(\frac{\Phi'_E}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_F} - R\frac{\partial R_E}{\partial \overline{\omega}_F} - \frac{\Phi'_F}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E}\right) \\
&= \left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right)^{-1} \left(\frac{\Gamma'_E - \mu_E \overline{\omega}_E}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_F} - R\frac{\partial R_E}{\partial \overline{\omega}_F} - \frac{\Phi'_F}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E}\right) \\
&= \left(\Phi'_F + \frac{\partial R_E}{\partial \overline{\omega}_F}\right)^{-1} \left(-\left\{R - 1 + \frac{\mu_E \overline{\omega}_E}{\Gamma'_E}\right\}\frac{\partial R_E}{\partial \overline{\omega}_F} - \frac{\Phi'_F}{\Gamma'_E}\frac{\partial R_E}{\partial \overline{\omega}_E}\right) \\
&< 0, \qquad (36)
\end{aligned}
$$

as long as $R > 1$. Therefore, the cost of curve decreases as n_E increases.

C The BGG Model

In this appendix, we provide equilibrium conditions of our BGG model. The key difference between our baseline model and the "BGG model" is that the model abstracts from the credit constrained FIs and that it is the entrepreneurs instead of the FIs that solve the profit maximization problem in the BGG model. Similarly to BGG (1999), the entrepreneurs in the BGG model solves their profit maximization problem subject to the participation constraint of the FIs that is specified as follows:

$$\left[\int_{\overline{\omega}_E(s^{t+1})}^{\infty} \overline{\omega}_E\left(s^{t+1}\right) dF_E\left(\omega_E\right) + \left(1-\mu_E\right)\left(\int_0^{\overline{\omega}_E\left(s^{t+1}\right)} \omega_E dF_E\left(\omega_E\right)\right)\right] R_E\left(s^{t+1}\right) Q\left(s^t\right) K\left(s^t\right)$$
$$\geq R\left(s^t\right)\left(Q\left(s^t\right) K\left(s^t\right) - N_E\left(s^t\right)\right), \tag{37}$$

where $\overline{\omega}_E\left(s^{t+1}\right)$ is the cut-off value in the credit contract between the entrepreneurs and the FIs and $R\left(s^t\right)$ is risk-free rate. Here, the FIs collect deposits from households and lend them to the entrepreneurs in a competitive manner. Consequently, similarly to BGG (1999), the FIs earn zero profit and do not accumulate their own net worth and only the entrepreneurial net worth plays a role in the financial accelerator effect. Apart from the structure of the credit market, the rest of the BGG model, including goods market, is the same as the baseline model.

The equilibrium conditions in the BGG model are given by equations (13), (15), (16), (19), (20), (22), (24), and the participation constraint that is specified above as well as following two equations that correspond to equations (9) and (26) in the benchmark model:

$$\begin{aligned} 0 = & \sum_{s^{t+1}} \Pr\left(s^{t+1}\right)\left(1-\Gamma_E\left(\overline{\omega}_E\left(s^{t+1}\right)\right)\right) R_E\left(s^{t+1}\right) \\ & + \frac{\Gamma'_E\left(\overline{\omega}_E\left(s^{t+1}\right)\right)}{\Phi'_E\left(s^{t+1}\right)} \Phi_E\left(s^{t+1}\right) R_E\left(s^{t+1}\right) - \frac{\Gamma'_E\left(\overline{\omega}_E\left(s^{t+1}\right)\right)}{\Phi'_E\left(s^{t+1}\right)} R(s^t), \text{ and} \end{aligned} \tag{38}$$

$$\begin{aligned} Y\left(s^t\right) = & \, C\left(s^t\right) + I\left(s^t\right) + G\left(s^t\right) \\ & + \left(\mu_E \int_0^{\overline{\omega}_E\left(s^t\right)} \omega_E dF_E\left(\omega_E\right)\right) R_E\left(s^t\right) Q\left(s^{t-1}\right) K\left(s^{t-1}\right) + C^E\left(s^t\right), \end{aligned} \tag{39}$$

with

$$C_E\left(s^t\right) \equiv \left(1-\Gamma_E\left(\overline{\omega}_E\left(s^t\right)\right)\right) R_E\left(s^t\right) Q\left(s^{t-1}\right) K\left(s^{t-1}\right).$$

Here, the first equation describes the first order condition for the entrepreneurs and the second equation describes the resource constraint in the economy.

D The Modified BGG Model

In this appendix, we provide equilibrium conditions of our modified BGG model. In this model, the participants of the credit markets are FIs and entrepreneurs. As in the BGG model, the FIs are not credit constrained. However, unlike the BGG model and like our baseline model, the FIs are monopolistic loan suppliers to the entrepreneurs. Not the entrepreneurs but the FIs maximize their profits. The entrepreneurs join the credit contracts only when the return from the credit contract is at least equal to their opportunity costs.

A type i FI makes loans of $Q(s^t)K_i(s^t) - N_{E,j_i}(s^t)$ to group j_i entrepreneurs. It finances the loan by collecting deposit from households with risk-free rate $R(s^t)$. From lending to a group j_i entrepreneur, an FI expects to earn the amount of $\Phi_E\left(\overline{\omega}_{E,j_i}(s^t)\right)R_E(s^{t+1})$ $Q(s^t)K_{j_i}(s^t)$.Thus, the FI optimally chooses $K_{j_i}(s^t)$ and $\overline{\omega}_{E,j_i}(s^t)$ to maximize their expected profit described as follows:

$$\Phi_E\left(\overline{\omega}_{E,j_i}\left(s^t\right)\right)R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right) - R\left(s^t\right)\left(Q\left(s^t\right)K_i\left(s^t\right) - N_{E,i}\left(s^t\right)\right). \quad (40)$$

Note that entrepreneurs face the participation constraint that is the same as that under the baseline model. A participation constraint of a group j_i entrepreneur is therefore given by

$$\begin{aligned}&\sum_{s^{t+1}}\Pr\left(s^{t+1}\right)\ R_E\left(s^{t+1}\right)Q\left(s^t\right)K_{j_i}\left(s^t\right)\left(1-\Gamma_E\left(\overline{\omega}_E\left(s^{t+1}\right)\right)\right)\\ &\geq\sum_{s^{t+1}}\Pr\left(s^{t+1}\right)R_E\left(s^{t+1}\right)N_{E,j_i}\left(s^t\right)\ \text{for}\ \forall j_i. \end{aligned} \quad (41)$$

Arranging the first-order condition gives the following condition for the optimal contract:

$$\begin{aligned}0 &= \sum_{s^{t+1}}\Pr\left(s^{t+1}\right)\left(1-\Gamma_E\left(\overline{\omega}_E\left(s^{t+1}\right)\right)\right)R_E\left(s^{t+1}\right)\\ &+\frac{\Gamma_E'\left(\overline{\omega}_E\left(s^{t+1}\right)\right)}{\Phi_E'\left(s^{t+1}\right)}\Phi_E\left(s^{t+1}\right)R_E\left(s^{t+1}\right)-\frac{\Gamma_E'\left(\overline{\omega}_E\left(s^{t+1}\right)\right)}{\Phi_E'\left(s^{t+1}\right)}R(s^t). \end{aligned} \quad (42)$$

This expression is exactly the same as that in the BGG model. Therefore, differences between the BGG model and the modified BGG model in terms of contents of credit contract, $K_{j_i}(s^t)$ and $\overline{\omega}_{E,j_i}(s^t)$, arise from the participation constraint: equation (41) for the modified BGG model and equation (37) for the BGG model.

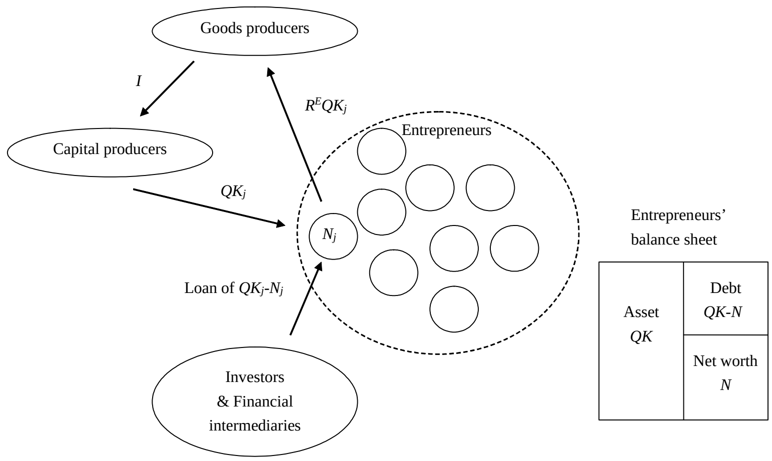

Figure A. Credit contract in the BGG model

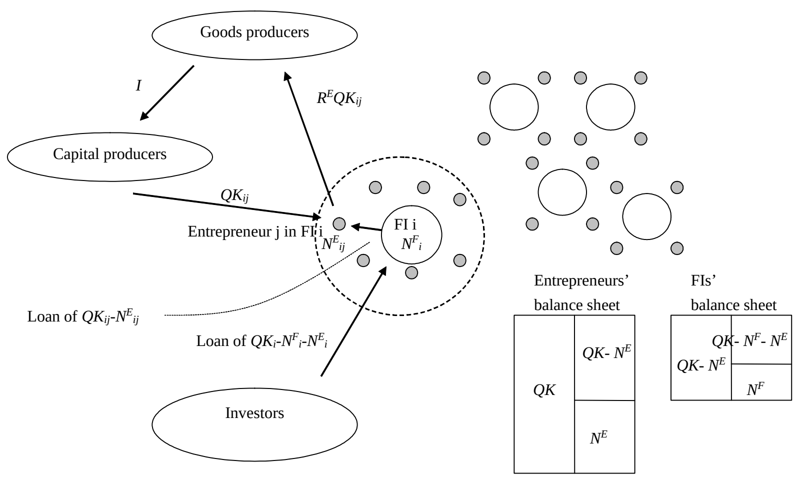

Figure B. Chained credit contracts in our model

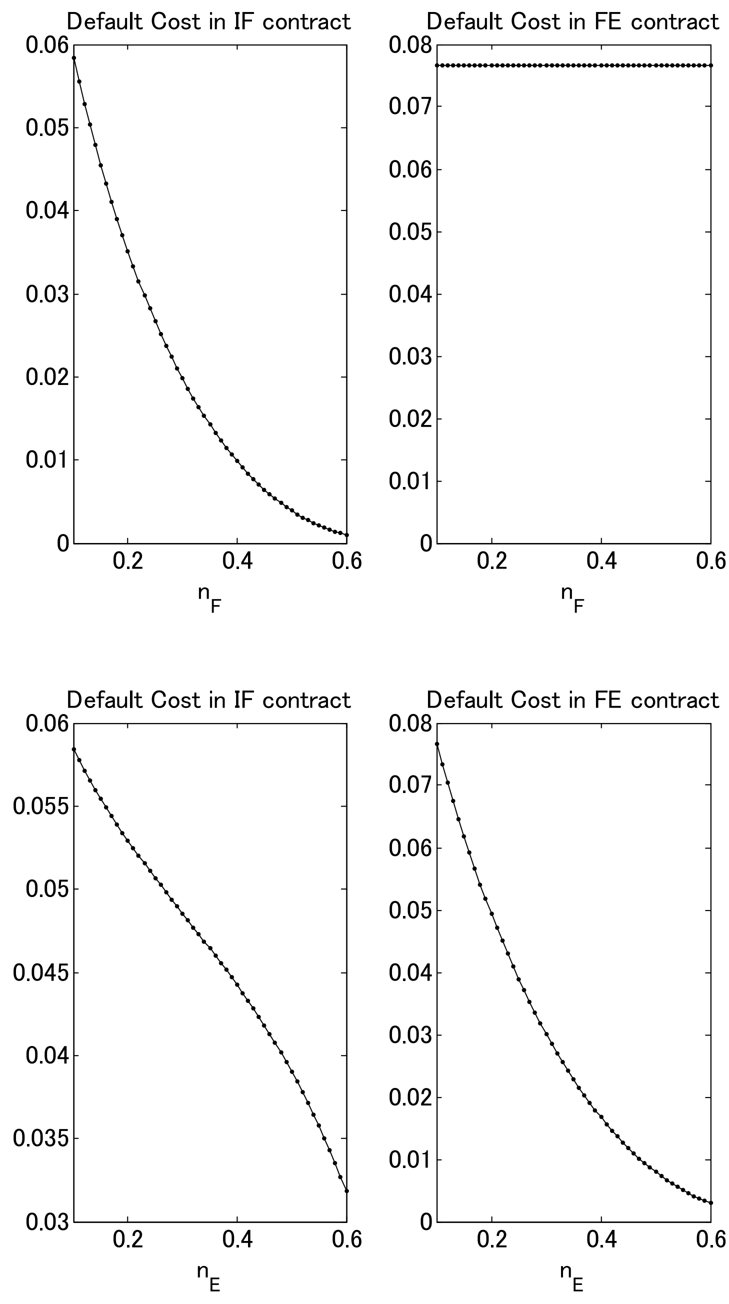

Figure C. Effects of borrowers' net worth on the expected default cost.

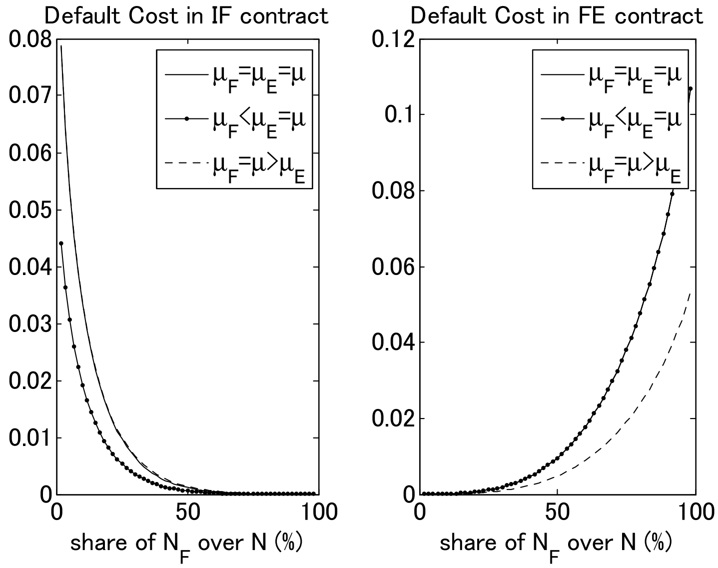

Figure D. Effects of net worth distribution and monitoring cost on default cost

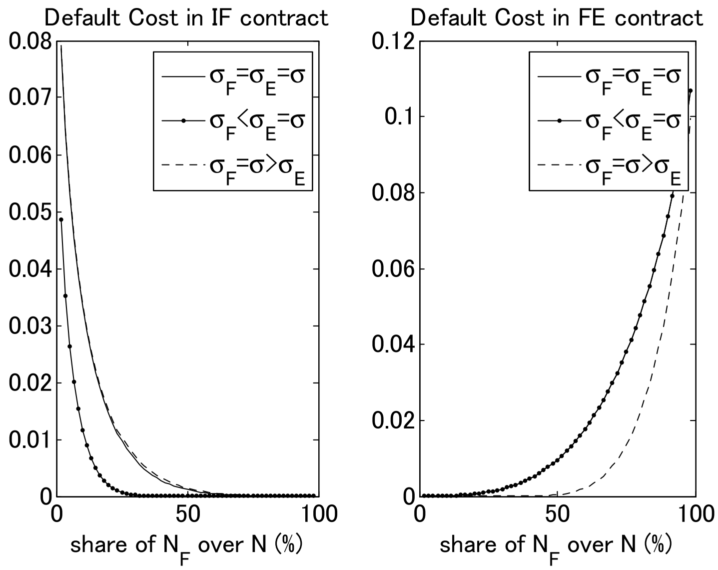

Figure E. Effects of net worth distribution and uncertainty on default cost